Learning Mathematics with tools

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Summary

This paper presents the most recent researches in Mathematics Education in Italy, dealing with the use of tools and technologies at school. Its origin is in the document of Unione Matematica Italiana (U.M.I., the association of school and university teachers of Mathematics), which has published a curricular project for school Mathematics from the 1st to the 12th grade. The paper is a part of the book that is presented at ICME as the collection of all the most recent Italian researches in Mathematics Education.

Introduction

Learning Mathematics with tools, which can be traditional or more advanced, is simpler than in a abstract way (called symbolic-reconstructive by the Psychologists): it is a perceptive-motor way of learning, because it is grounded in actions, perceptions, and reactions due to the feedback received in the use of tools (Antinucci, 2001).

The methodologies, the activities, the curricular sequences used by the teacher are different from a traditional way of teaching, based only on frontal lessons, and the role of teacher is very important not only in realising students' activities, but also in planning them.

This paper is related with the theme of the DG20, because it offers an overview on Italian recent researches on the use of tools in teaching and learning Mathematics. So, it is aimed at giving a possible answer to some of the questions asked by the DG:

1. About the Theme (C) Tools and Technologies:
   - What are appropriate/meaningful uses of technology for upper secondary mathematics?
   - What can be the different roles of tools and technologies in the mediation of learning?
   - How can the use of tools and technologies influence students' cognitive processes?
2. About the theme (D) Curriculum:
   - Can new theoretical trends influence school curricula?
   - What are the new curricular trends recently developed in different countries?

Tools in Mathematics Education: recent Italian trends

Many research studies have been carried out in the last few years concerning learning with tools, i.e. learning in an environment that is richer than that of the standard paper and pencil. The empirical and theoretical studies of Italian researchers have led to the elaboration of the idea of mathematical laboratory (e.g. Mariotti, 2002; Chiappini & Reggiani, 2003; Arzarello, Paola & Robutti, 2002; Arzarello, Andriano et al., 2000; Bonotto et al, 2002) and have been collected by the committee appointed by U.M.I. for the production of new curricula between years 2000 and 2003. In the following we shall quote wide excerpts of the document prepared by the U.M.I.[1] committee together with some further elaborations by the authors. The U.M.I. text has been partially adopted in the official documents of the Italian Ministry of Education[2].

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[1] http://www.dm.unibo.it/umi/italiano/Didattica/2003/secondaria.pdf

[2] http://www.istruzione.it/news/2002/allegati/sperimentazione/profilo_terminale.pdf

'A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments). We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students). It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning can not be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities' [1] (p. 32).

This introduction is accompanied by an annotated list of exemplary tools, taken from everyday experience, and advanced technological tools as well.

'Poor materials. For example, working with transparent slides, the crease of paper, the use of pins, grid paper, should not only be considered an activity specifically designed for pupils of primary schools, but it could be a meaningful starting point for mathematical activities at different levels. Furthermore, the use of poor materials, made by the students themselves, represents a significant activity, in the spirit of the Renaissance workshops' [1] (p. 32).

Some research groups have designed, carried out and analysed teaching experiments (usually in grades 1st-8th) concerning either mathematical objects built with poor materials (e.g. Facenda et al., 2002) or everyday objects analysed through mathematical lenses, in order to elicit the implicit mathematical ideas (e. g. cardboard area units, an informational booklet issued by "Poste Italiane", cotton tips and square napkins in Bonotto & Ceroni, 2003; supermarket receipts in Bonotto, 2001; TV schedule in Bonotto, 2003). In the latter case the introduction of cultural artefacts from out-of-school experience aims at creating a new tension between school mathematics and everyday-life knowledge. The shift from the study of everyday gears to mathematical objects like circles has been studied by Bartolini Bussi, Boni & Ferri (to appear). The analysis of a traditional mathematics object like the graduated ruler for the introduction of decimal numbers and their properties is in Bonotto et al. (2002). The ruler suggests the shift towards tools that have been created and constructed throughout history for specific mathematics purposes, such as mathematical machines.

'*The mathematical machines. The possibilities offered by the mathematical machines, of manipulating objects physically, as in the case of machines generating conics, often induces exploration and construction of mathematical meanings, different but not less interesting than the one offered by Dynamic Geometry Software'* [1] (p. 32).

A mathematical machine (related to the geometry field) has a basic aim, that does not depend on the practical use (if any) of the artefact. It aims at forcing a point, a line segment or a plane figure (supported by a suitable material support that makes them visible and touchable) to move or to be transformed according to a mathematical law that has been determined by the designer. A very large collection of mathematical machines, used both in exhibition and in the classroom, is in Modena (www.mmlab.unimore.it). The most well-known mathematical machine is the compass (see Bartolini Bussi & Boni, 2003; Bartolini Bussi, Boni & Ferri, to appear), that is the ancestor of many curve drawing devices (Bartolini Bussi, 2001; several papers by Pergola et al.). Another class of mathematical machines is given by perspectographs (Bartolini Bussi, Mariotti & Ferri, to appear) that are related to the ancient 3d theory of conics (Bartolini Bussi, to appear). An international review of ancient instruments that can be used in the classroom is in Bartolini Bussi (2000). Besides geometrical ones, there are also instruments for arithmetic (for a didactical analysis of abacus see

Bartolini Bussi & Boni, 2003; Ferri 2002, Betti & Canalini 2002). The theoretical framework of the above experimental studies in based on the Vygotskian idea of semiotic mediation.

'The DGE: Dynamic Geometry Software. During the last few years, the teaching of Geometry has been supported by the introduction of Dynamic Geometry Software, that are microworlds designed for specific educational tasks. They allow students to experience, to explore and to observe, in order to look for invariants, patterns and regularities, and to formulate conjectures, and to test them in the software itself. In such a kind of interaction with the microworld, the student can meet the knowledge embodied in the software, and he can then construct a proper geometric knowledge. From this activity, the student can pursue a more theoretical knowledge, namely the proof, as the activity aimed at justifying why a certain property holds in a given theory' [1] (p. 32).

This is a very popular field. Many Italian researchers have published papers concerning classroom experiments with Cabri. The approach to the definition of geometrical figures is considered by Pesci (2000), with possible extension to primary school pupils too. However most researchers are interested in analysing the processes of conjecturing and proving (Olivero, 2001, 2003; Olivero et al. 2002, Arzarello et al., 1999a, 1999b; Olivero, Paola & Robutti, 2001). In particular, Arzarello (2000) describes the learning of proof as a long process of interiorisation, through specific and complex mental dynamics of pupils, from perceptions and actions within technological environments towards structured abstract mathematical objects, embedded in a theoretical framework: the main issues in the analysis of students' performances consist in metaphors, deictics, mental times, narratives, functions of dragging, abductions, linear vs. multivariate language and so on, to be used within an embodied cognition perspective. Arzarello, Olivero et al. (2002) offer a fine grain analysis of the process of dragging in conjecturing and proving. Mariotti (2001a, 2001b) analyses the teacher's role: starting from a Vygotskian perspective, attention is focussed on the social construction of knowledge and on the semiotic mediation accomplished through cultural artefacts. The functions of specific elements of the software are described and analysed as instruments of semiotic mediation used by the teacher in classroom activities.

Another group of papers consider the measuring process in the Cabri microworld. Olivero & Robutti (2001a, 2001b, 2002) study the shift from perception to theory and back again fostered by the measurement tool and the effectiveness of this shift in the construction of a proof, after the conjecturing phase.

Two papers (Laborde & Mariotti, 2002; Mariotti, Laborde & Falcade, 2003) study the approach to the concept of function in the Cabri microworld. The authors analyse experiments in secondary school where the dynamic features provide a basic representation of both variation and functional dependency.

The comparison between Mascheroni geometry and Cabri geometry is studied by Galoppin P. & Zuccheri L. in secondary school. The difficulties met by foreign students in the Cabri microworld are studied by Rocco (2000).

Accascina & Margiotta in a set of papers discuss interesting problems on the geometry of triangles to be used with secondary school students. Bernardi (2003) discusses some epistemological issues related to dragging in Cabri. The role of new technologies in the teaching of geometry is analysed in a context of teacher training, by De Petro et al (2003) and Zuccheri (2003).

For more than ten years IRRE Emilia Romagna has been gathering and spreading didactical innovative experiences with Cabri Géomètre at different school level and promotes exchanges and discussions on various issues concerning the use of Cabri in the real context. At the following URL, http://www.fardiconto.it, teachers, students and researchers can find different resources (discussion forum, publications, experiences, new didactic situations …) provided by IRRE Emilia Romagna which can be very useful to favour a didactical use of Cabri in class.

'The CAS: Computer Algebra Systems. In the teaching of algebra and calculus a primary role is played by the CAS, which have different integrated environments, generally the numerical, the graphical, the symbolic and the programming. The introduction of CAS in the teaching of algebra and calculus permits to circumscribe the use of symbolic calculation with paper and pencil only to

simpler cases, in order to let more complex calculations to be done by the student with the aid of the software. From a didactical point of view, we can have a double advantage, because the student is free to concentrate on the meaning of the calculation, if he can devolve the difficult one to the CAS. Even the CAS, like the DGE, offers to the student different environments in which he can explore and make conjectures, in order to construct the meaning of mathematical objects.

Last, but not least, the programming language offered by CAS is particularly useful for consolidating the concept of function, variable, input and output values, and of data collection (list, array, matrix, …)' [1] *(p. 33).*

Many researchers dealt with the theme of the didactical use of computer algebra systems but most studies on this topic were carried out using graphic–symbolic calculators (see below).

The most widespread CAS in Italy at the didactic level is *Derive*, in a version for Windows, in that it is particularly simple to use and, moreover, it is available in Italian. There are many didactical proposals and suggestions in school text-books, but less numerous are research activities. The use of *Derive* as a symbolic manipulator in the phase of approach to algebra is studied in Reggiani (2002a), where some aspects of the problem of writing, reading and processing algebraic expressions with *Derive* are analysed, focusing on abilities required and promoted by this software and on differences with paper and pencil.

In other studies the use of *Derive* as a help for the formulation of conjectures and as a tool for their verification is dealt with. This aspect is proposed in Reggiani (2002b) about some questions of divisibility and other problems whose generalization requires algebraic competencies, and in Reggiani (2000) about the study of functions depending on parameters. The mediation role of the software in the construction of the meaning of parameter through the observation of graphs and algebraic manipulation is pointed out.

The strategies used in solving geometric problems depending on the tools which can be used are studied by Accascina (2001). The author compares strategies used by students working with or without *Derive* and analyses the pros and cons of the use of *Derive* in solving geometric problems in the school final examinations.

A more general question is proposed in Impedovo (2002) who, starting from the hypothesis that students have at their disposal all the time (during classes, while studying at home and for any assignment and examination) a Computer Algebra System or, more generally, mathematics software like *Derive*, MAPLE, MATHCAD, or graphic and symbolic calculator, examines in which way contents, teaching of mathematical objects, problems, exercises and finally evaluation instruments should be modified.

The teaching of algebra and arithmetic (in previous years) is approached also by means of specific microwolds. In the domain of Arithmetic two papers (Bottino, & Chiappini, 2002; Bottino, 2000) deal with the relationship between the use of microworlds and the construction of educational environments able to foster teaching and learning processes in this domain. In the domain of Algebra some papers (Chiappini, Pedemonte, & Robotti, 2003; Mariotti, & Cerulli, 2001; Mariotti, & Cerulli, 2002; Cerulli, & Mariotti, 2003) deal with the role of specific microworlds in the learning of Algebra according to an innovative educational approach in which algebraic manipulation is viewed as a demonstration of the equivalence of two forms of expressions. It is important to observe that the microworlds described in these papers (the microworlds of ARI-LAB-2 and the microworld named Algebrista) are developed by the authors of these papers.

'The spreadsheets. Spreadsheets, developed as tools for business and financial calculation, not for educational purposes, have various applications in the school, particularly related to statistics (data collection, organisation, graphical representation, ...) and probability. But another fundamental use of spreadsheets is the one related to modelling, representing functions and even geometric transformations' [1] (p. 33).

Italian researchers have also studied the role of spreadsheet in the construction of mathematical knowledge. In particular a paper (Arzarello, Bazzini & Chiappini, 2002) analyses the role of a spreadsheet in structuring a didactic space-time of production and communication (SP) able to *favour the production and interpretation of formulae in the approach to algebra and the use of variables and parameters in modelling complex situations. Through a comparison between the SP structured with the mediation of a spreadsheet and the traditional SP based on the use of paper and pen, this publication suggests a model to analyse algebraic thinking and to design didactic situations apt to build up a genuine algebraic knowledge.*

Another paper (Lemut, 2003) analyses the role of a spreadsheet in supporting and creating the conditions for Systemic Thinking development. In this paper Systemic Thinking is considered as a general philosophy that, by suggesting a "thinking globally, but acting locally" approach, can represent a major paradigm shift in how we view the world.

'The symbolic-graphic calculators. All the support environments offered by the software previously described, can be found in the symbolic-graphic calculators, which can be used with more flexibility and simplicity, both for the space occupied, and the time utilised (to move students from a classroom to a laboratory). Many of these calculators offer the possibility to connect with a sensor, to measure a physical quantity and to collect data in real time. This modality is of particular importance, as regards the possibility of describing a phenomenon in mathematical language, thus obtaining a model.' [1] (p. 33).

Various studies were carried out in recent years, about the introduction of mathematical concepts through experiments involving perceptual-motor activities, as for example body motion or the motion of objects, as toys, balls and so on. The didactical aim of these studies is the construction of the meaning of graphs and number tables related to the motion activity, in order to avoid the most frequent misconceptions witnessed in the literature. The research aim is to analyse students' cognitive processes, in terms of a detailed outline of gestures, metaphors and language. These studies started from an initial enquiry on students' performances, analysed through the theory of *embodied cognition*. Their cognitive activities, revealed by words and gestures, are crucial for the genesis of their mathematical understanding. Specifically, the so called *grounding metaphors* and *fictive motions* are cognitive pivots which trigger and support the transition from empirical and perceptive facts to a more theoretical frame (Arzarello & Robutti, 2001).

Different research studies have been carried out within this framework.

A first set of studies aimed at the construction of the concept of function as a tool for modelling motion with 9th graders (Ferrara & Robutti 2002a; Ferrara, & Robutti 2002b; Arzarello, Pezzi & Robutti, 2003). The topic has been explored deeply through various activities, also in environments outside school, as for example Luna-park. These activities can help and support students in a meaningful approach to algebraic rules, symbols and relationships. The focus is on developing the symbol sense, as well as interconnecting syntactic and semantic aspects.

Another set of studies carried out with symbolic-graphic calculators, refers to the construction of the concept of integral, starting from approximate measures of areas of figures. This study, based on the didactical aim of introducing Calculus concepts grounding on David Tall's cognitive roots, has the research aim of analysing students' cognitive processes, in terms of the mediation of technology and gestures, metaphors and language. A long teaching experiment in upper secondary school (11th-12th grade) is presented (Robutti & Sabena, 2003). The study specifically analyses the passage from finite sums to infinite ones, with the mediation of technology (Robutti, 2003). The other side of the coin is the concept of derivative, constructed from the local slope of the graph of a function: a fine study based on the use of Zoom in the symbolic-graphic calculators was carried out in a PhD Thesis and the papers related to it (Maschietto, 2002; Accomazzo & Maschietto, 2002).

Here the list of studies related to the UMI document ends. In the same spirit we may add other tools that have been used in the classroom to enhance mathematics activity.

Videotapes may be used in the classroom to foster metacognitive activity: students may observe themselves at work and reflect on their own processes.

In the paper (Furinghetti, Olivero & Paola, 2001) videotapes are used to encourage students to reflect on their reasoning. The same approach appears in the paper (Olivero, Paola & Robutti, 2002). Videotapes may be used also by teachers and researchers to analyse students' performances.
There are some experiences of e-learning, as for example the one described in the papers (Iozzi 2002, Osimo 2002), which presents an undergraduate mathematics course within a business administration course. The course, which is part of a three-years "Degree in Economics of International Markets and New Technology", deals with topics of pre-calculus, calculus, linear algebra. The use of technological tools seems to be essential for today's learning methodologies. The paper offers a challenge to the possibility of changing both the ways of teaching and the contents of a mathematical course.
The introduction of *tools for textbook analysis* in the context of teacher training is studied by Formica et al. (2001).
It seems important to remind that most of the studies reviewed (the ones concerned with classroom activity) focus not only on the features of the tool, but also on the quality of interactions (student–instrument; student–student; students–teacher). This shared idea has been taken also in the U.M.I. document:
'The construction of meaning with a methodology based on the Mathematics laboratory is strictly connected with the social interaction of students, during an activity, carried out in small groups work. During the group activity, the students can share the process of conceptualisation, through a collaborative or cooperative interaction. After the group activity, it is hopeful that a class discussion, led by the teacher, permits students to share the results of the groups. A mathematical discussion consists of a social interaction aimed at the construction of a common knowledge in the classroom, shared by all students' [1] (p. 34).

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